

Calculators are not allowed

Answer the following questions (all items are weighted equally at 5 points each):

1. (a) Use the definition of the limit to show that

$$\lim_{x \rightarrow -\frac{1}{2}} (6x + 1) = -1.$$

- (b) State the intermediate value theorem. Show that the equation

$$x^2 \sin \pi x + 2x^3 - 1 = 0$$

has a real solution.

2. (a) Find the vertical and the horizontal asymptotes (if any) of the graph of the function

$$f(x) = \frac{3x^2 |x - 1|}{x^3 - 1}$$

- (b) Use the definition of the derivative to show that, if f and g are differentiable functions at $x = a$ and $f(a) = g(a)$, then

$$f'(a) - g'(a) = \lim_{x \rightarrow a} \frac{f(x) - g(x)}{x - a}.$$

3. (a) Let

$$f(x) = \begin{cases} \frac{\sin x}{x} & , \text{ if } x < 0 \\ x + 2 & , \text{ if } 0 \leq x < 2 \\ \frac{4|x - 2|}{x - 2} & , \text{ if } 2 \leq x \end{cases}$$

Classify the discontinuities of f as removable, jump, or infinite.

- (b) Find $f'(x)$, if

$$f(x) = \frac{x + \tan x}{\sin x + 5x^2 + 4}$$

4. Let $f(x) = x^{\frac{2}{3}}(x - 1)$. Find the x -coordinate of the points on the graph of f (if any) at which

(a) the graph of f has a horizontal tangent.

(b) the graph of f has a cusp.

(Good Luck)